Collisions

Teacher's Guide

UK Space Agency

The UK Space Agency is at the heart of UK efforts to explore and benefit from space. It is responsible for all strategic decisions on the UK civil space programme and provides a clear, single voice for UK space ambitions.

The Agency is responsible for ensuring that the UK retains and grows a strategic capability in the space-based systems, technologies, science and applications. It leads the UK's civil space programme in order to win sustainable economic growth, secure new scientific knowledge and provide benefits to all citizens.

ESA

From the beginnings of the 'space age', Europe has been actively involved in spaceflight. Today it launches satellites for Earth observation, navigation, telecommunications and astronomy, sends probes to the far reaches of the Solar System, and cooperates in the human exploration of space.

Space is a key asset for Europe, providing essential information needed by decision-makers to respond to global challenges. Space provides indispensable technologies and services, and increases our understanding of our planet and the Universe. Since 1975, the European Space Agency (ESA) has been shaping the development of this space capability.

By pooling the resources of 22 Member States, ESA undertakes programmes and activities far beyond the scope of any single European country, developing the launchers, spacecraft and ground facilities needed to keep Europe at the forefront of global space activities.

National Space Academy

Established in 2011 and led by the National Space Centre, the National Space Academy is now the UK's largest space education and skills development programme for secondary and further education. Its team includes some of the country's best science teachers, project scientists and engineers who deliver masterclasses and intensive teacher training for thousands of students and teachers across the UK every year. Internationally the Academy works extensively with the European Space Agency, the UAE Space Agency, and it also leads the UK's ongoing space education and skills development work with China.

Astro Academy Principia

A unique education programme developed by the UK's National Space Academy for the UK Space Agency and ESA (European Space Agency), Astro Academy: Principia uses a suite of demonstrations filmed by ESA astronaut Tim Peake aboard the ISS during his six month Principia mission to explore topics from secondary physics and chemistry curricula. The programme is made up of stand-alone teaching films, downloadable video clips, downloadable files that can be used with the free-to-use dynamical analysis software programme "Tracker", written teacher guides and links to more than 30 further teaching activities.

Principia

Tim's mission to the International Space Station, called 'Principia', used the unique environment of space to run experiments as well as try out new technologies for future human exploration missions. Tim was the first British ESA astronaut to visit the Space Station where he spent six months as part of the international crew.

Collisions and Momentum

Introduction

Tim Peake's launch into space from Baikonur Cosmodrome in December 2015 was a classic demonstration of fundamental principles involving the conservation of momentum – essential in understanding how all rockets work. The conservation of momentum – and understanding the consequences and applications of this principle – is one of the most powerful contextual tools in physics and engineering. In this teacher guide, simple classroom demonstrations are used to illustrate the fundamental principles relating to both elastic and inelastic collisions. Tim's orbital demonstrations enable students to extend this further through visualisation of momentum modelling in multiple directions and a comprehensive suite of space applications. Extension questions include contexts ranging from the formation of the Moon to the discovery of hidden water on the planet Mars and the development of new technologies in space propulsion systems.

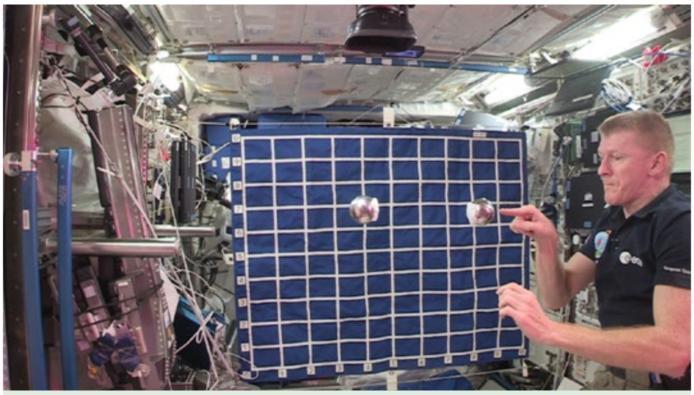
Tim's orbital demonstrations

Clips:
V1 Two equal mass head on side view
V2 Two equal mass head on above – drift
V3 Oblique collision

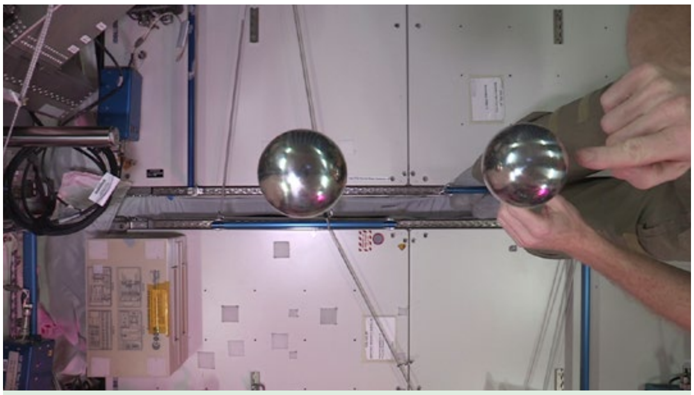
Microgravity gives a superb environment for demonstrating aspects and consequences of conservation of momentum, elastic/inelastic collisions and associated applications because we can conduct demonstrations that model in one, two and three dimensions vectorially in a way that is much more challenging here on Earth.

Tim conducted various types of collisions between balls of different masses, with some collisions resulting in the balls sticking together through the use of velcro.
For all the collisions, filming was conducted using synchronised side-view and top-view cameras to aid multi-dimensional vector analysis by students and teachers.

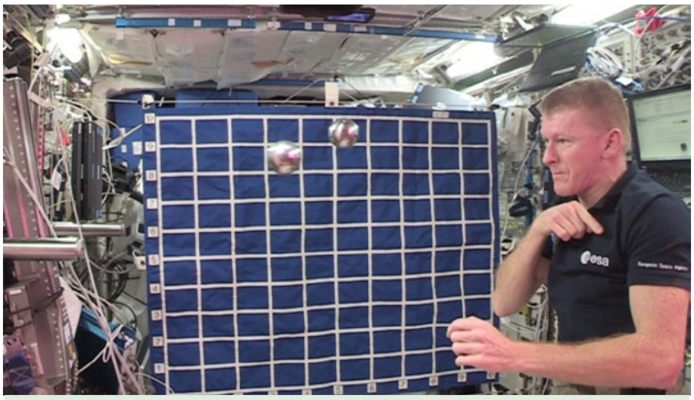
1) Collisions between balls of equal masses



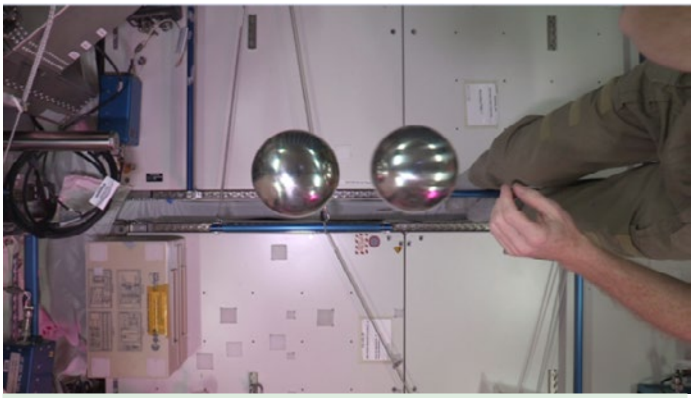
Side view of Tim conducting collision experiment using balls of equal mass – setting up the collision using his finger like a snooker cue



Above: Top view of the same collision



Side camera view post-collision: we can see how the target ball is moving off to the left as expected but the impacting ball now has an upwards component to its motion – because the centres of mass of each ball just before the collision were not lined up exactly. Doing so in microgravity is very challenging!



Top-view of same impact – we can see in this view how the impacting ball is moving upwards and therefore slightly larger in the field of view of the top camera.

In classroom experiments, collisions are generally limited to a one-dimensional analysis (in the case of balls suspended on long strings) or two dimensional (in the case of snooker ball type collisions).

Effect of oblique collisions

If the balls' centres of masses are not along the line of translation of the inbound ball at the moment of impact then we have an OBLIQUE collision.

When this happens momentum is still conserved – but we need to model this in multiple directions.

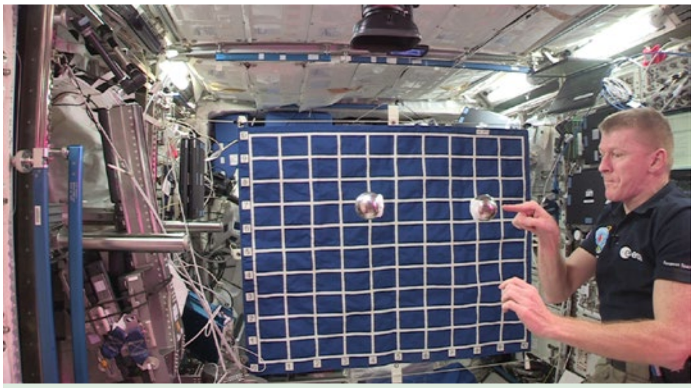
Collisions between the two balls of equal masses – momentum modelling

In all of these collision events we can clearly see that the sum of the final post-collision momentum components in each of the horizontal and vertical planes does indeed appear to add up to the initial pre-collision momentum components.

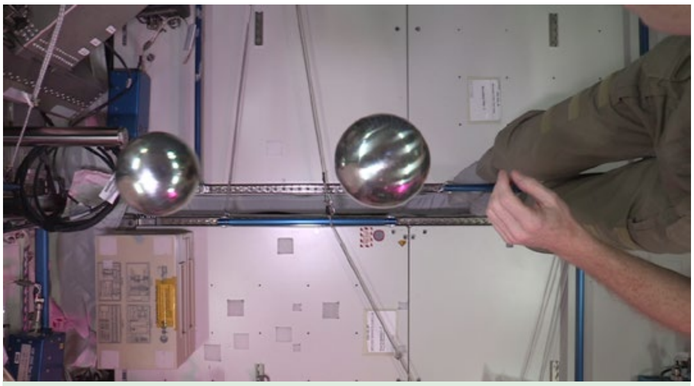
To check this fully, we would need a third camera view, positioned to look along the inbound ball trajectory plane, i.e. on the "left hand side" of the side view camera frame, looking towards Tim.

2) “Sticky” collisions – velcro covered balls, translation and rotation

Tim conducted several collision demonstrations using balls of the same mass which stuck together post-collision due to the velcro tabs placed on both balls. Once again, the top camera and side camera views have been synchronised for analysis.



Side view – before collision



Top view – before collision

Views after the two-ball collision

We can see once again the importance of the position of the centre of masses of both balls at the moment of impact, in relation to the direction of translation of the inbound ball.

Clips:
V4 Inelastic large spin
V5 Inelastic little spin
V6 Inelastic collision top view

The collisions between the velcro-clad balls were oblique, the lines of translation of the inbound ball did not intersect the centres of masses of both balls at the point of collision.

Post-collision, we see that the motion of the two—ball assembly had two major components:

- TRANSLATION of the centre of mass of the system
- ROTATION of the system about its centre of mass

Whenever we have an inelastic collision which is oblique, we will get a combination of translational and rotational motion arising post-collision. Purely translational motion will only arise if the collision is NOT oblique – i.e. if the centres of masses of both balls at the point of collision lines up with the line of translation of the inbound ball.

Kinetic energy modelling in the oblique sticky collisions

Pre-collision

- The impactor ball had translational kinetic energy

Post-collision

Some of the kinetic energy of the inbound ball was lost in the collision process due to adhesion of the velcro, sound wave production, and deformation of the velcro.

Post-collision, the two-ball assembly had traditional TRANSLATIONAL kinetic energy given by the familiar $1/2mv^2$ formulation

The two-ball assembly also had ROTATIONAL kinetic energy given by $\frac{1}{2}I\omega^2$

- I is the MOMENT OF INERTIA of the post-collision configuration about a specified axis of rotation
- ω is the angular velocity of rotation about the specified axis

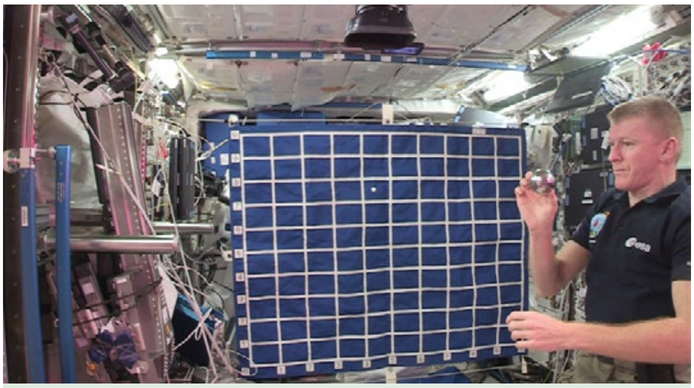
For multiple rotational axes, each will contribute towards the total kinetic energy of rotation through application and summation of the above rotational kinetic energy formulations.

3) Collisions between balls of unequal mass

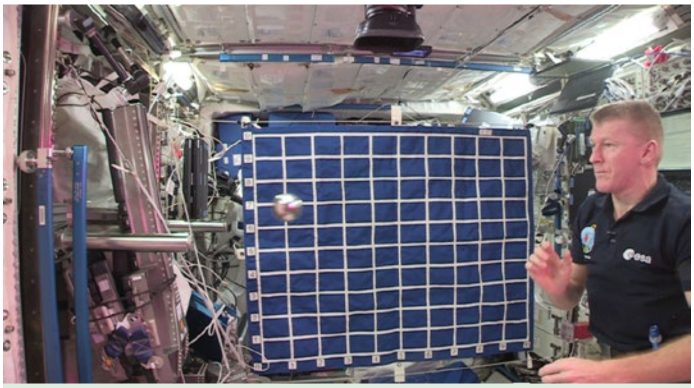
Tim conducted several elastic collisions between a stationary small ball and a large moving ball – and vice versa.

Clips:
V7 Large mass into small mass
V8 Large mass into small mass from above
V9 Small mass into large mass

a) Large ball impacting stationary small ball

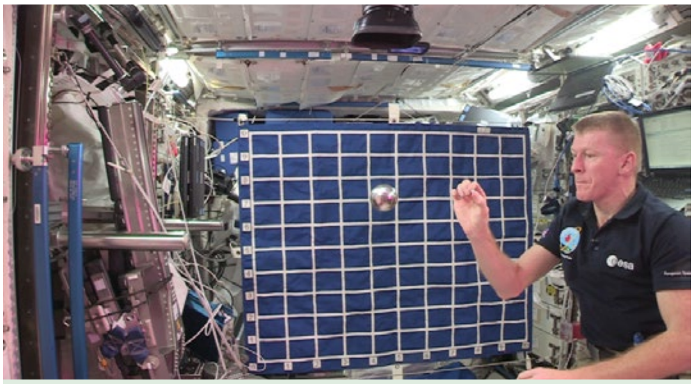


Stationary small ball being hit by large moving ball – before impact

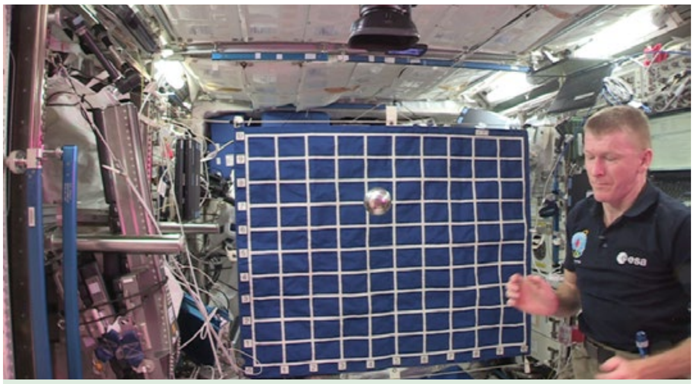


After impact - the moving large ball hardly changes in its state of motion whilst the small target ball has shot away with a high velocity towards the top left

b) Small ball impacting stationary large ball



Before impact



After the impact, we see that the larger ball hardly moves at all whilst the small inbound ball ricochets off with almost the same speed as it hit the target – but in a very different direction! It's moving in the direction that Tim is looking – downwards.

In all of the collisions between the balls of unequal masses, we see momentum conservation in three dimensions - momentum analysis in x, y and z directions show that momentum is conserved.

More detailed mathematical modelling and analyses of collision velocities is given in the ground-based experiment section.

Ground Based Experiments

Impacting balls

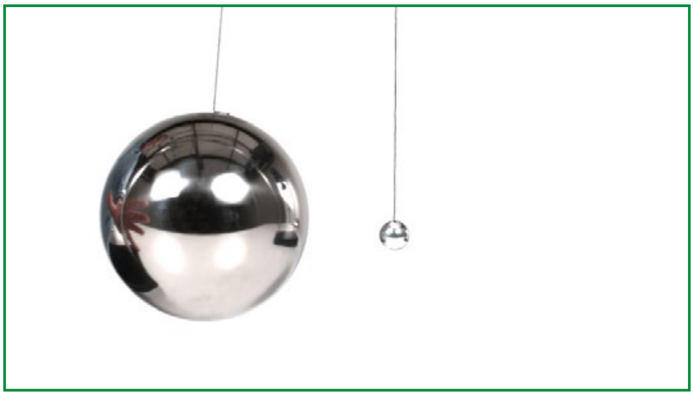
Curriculum Links:

- Collisions
- Conservation of momentum
- Kinetic energy

Key Stage: 4 and 5

Equipment List:

- Two clamp stands
- Additional metal rod
- Two bosses
- Two large identical mass aluminium balls
- One metal ball of much smaller size and mass
- Cotton thread
- Tape



Procedure:

Making the colliding balls

- Attach the bosses near the top of the two clamp stands and use these to hold the metal rod between the two stands.
- Cut two lengths of cotton to a length of 30 cm. Tape one piece of cotton to the top of one of the larger spheres and repeat with the other sphere.
- Tie the free end of each piece of cotton around the rod and use tape to adjust the length of each piece until the two spheres are touching at their centres.
- Repeat this procedure with a large ball and the smaller ball for the second demonstration.

Demonstrating 1-D collisions

- Carefully pull one of the larger balls back a distance of around 5cm. Make sure the other ball is stationary and the cotton thread is taught before releasing. Carefully release the ball allowing it to collide with the stationary ball of equal mass and observe what happens.
- Repeat this procedure with two balls of different mass, starting with pulling the smaller ball up and releasing it and following with pulling the larger ball up and releasing it. Again observe what happens.

Expected results

Two balls of equal mass:
The inbound ball should stop completely, transferring all of its kinetic energy to the outgoing ball which should move off at the same velocity. Both momentum and kinetic energy should be conserved (an elastic collision).

Consider momentum.

$mv_1 + 0 = 0 + mv_2$ showing $v_1 = v_2$

Smaller mass ball impacting larger mass ball:
The smaller ball should rebound off the more massive ball with almost all of its inbound speed maintained. The larger ball should recoil very slightly, but little momentum should be transferred to the larger mass ball.

Consider momentum.

$mv_1 + 0 = mv_2 + MV_2$
$V_2 = m(v_1 - v_2)/M$
If $M >> m$ then V_2 is small.

Larger mass ball impacting smaller mass ball:
The larger mass ball should be slowed down only slightly after impact while the smaller mass ball will move off in the same direction as the larger mass ball with very high velocity. While only a small fraction of the kinetic energy of the larger mass ball is transferred to the smaller mass ball, the difference in masses leads to a significant gain in velocity for the smaller mass ball.

$MV_1 + 0 = mv_2 + MV_2$
$v_2 = M(V_1 - V_2)/m$
If $M >> m$ then v_2 is large.

Modelling the collisions in more detail using conservation of kinetic and potential energies: mathematical modelling of elastic collisions between moving ball and stationary target

Before collision:

- Mass M_1 moves at velocity U
- It hits a stationary target of mass M_2

After collision:

- Mass M_1 moves at velocity V_1
- Mass M_2 moves at velocity V_2

From the conservation of kinetic energy and the conservation of momentum equations, it can be shown that, after the collision:

- **$V_1 = U \times (M_1 - M_2) / (M_1 + M_2)$ Equation 1**
- **$V_2 = U \times (2M_1) / (M_1 + M_2)$ Equation 2**

What if $M_1 << M_2$?
In this case, equation 1 will approximate to a value of V_1 that is nearly the same as U – but in the opposite direction. The small inbound ball rebounds with a velocity only slightly reduced. Equation 2 shows that the large target ball moves post-collision with a very slight velocity.

What if $M_1 >> M_2$?
In this case, equation 1 shows that the inbound ball velocity is only slightly reduced post-collision. Equation 2 shows that the small target ball shoots off with a very high post-collision velocity – nearly twice the velocity of the inbound large impacting ball.

What if $M_1 = M_2$?
In this case, the inbound ball stops completely and all its kinetic energy is transferred to the target ball, which moves at the same velocity post-collision as the inbound ball was moving pre-collision.

Extension – proving the collision equations:

From conservation of momentum:
- $M_1U = M_1V_1 + M_2V_2$ (1)

From conservation of kinetic energy (and after doubling both sides of the equation):
- $M_1U^2 = M_1V_1^2 + M_2V_2^2$ (2)

If we take the conservation of momentum equation (1) and rearrange to place all the M_1 terms on one side, we get:

$M_1(U-V_1) = M_2V_2$ (3)

We now do the same process for the kinetic energy equation (2) – rearranging to place all the M_1 terms on one side:

$M_1(U^2-V_1^2) = M_2V_2^2$ (4)

Equation 4 can also be expanded out and re-written as:

$M_1 (U+V_1)(U - V_1) = M_2V_2^2$ (5)

And if we now divide the energy equation 5 by the momentum equation 3, we obtain a simple expression relating U, V_1 and V_2:

$U + V_1 = V_2$ (6)

We can now use this expression for V_2 and substitute it into our original conservation of momentum equation (1) to yield the following:

$M_1U = M_1V_1 + M_2 (U+V_1)$

Dividing through by M_1 now gives us an expression for U:

$U = V_1 + (U+V_1) M_2/M_1$

Which can be rearranged to give:

$U(1-M_2/M_1) = V_1 (1 + M_2/M_1)$

Rearranging to make V_1 the subject of the equation :

$V_1 = U (1 - M_2/M_1)/ (1+ M_2/M_1)$

And finally, a little multiplying and dividing by factors inside and outside the brackets of M_1 and $1/M_1$ gives us:

$V_1 = U (1/M_{1)}) (M_1 - M_2)) / (1/M_{1)}) (M_1+M_2)) =$ $\mathbf{U (M_1 - M_2)/(M_1+M_2)}$ as required

If we now take this expression for V1 and substitute it into equation (6):

$V_2 = U+V_1 = U + U (M_1-M_2)/(M_1+M_2)$

And writing both terms on the right hand side as having (M1+M2) factors, this yields:

$\mathbf{V_2 = U \times 2M_1/(M_1 + M_2)}$

Newton's Cradle

Curriculum Links:

Conservation of momentum

Key Stage: 4 and 5

Equipment List:
- Newton's cradle

Procedure:
- Make sure the Newton's cradle is resting on a stable, level surface.
- Pull back one of the end balls to a distance of about 3cm and release it. Observe what happens. Now pull back two balls at the same time and release. Repeat this for three and four balls.

Expected Outcomes:
When one ball is raised and released it strikes the remaining balls with a certain impact velocity and therefore momentum. In a collision, momentum must be conserved. However when two objects have the same mass, kinetic energy is also conserved and maximum transfer of momentum occurs. The incoming object will stop dead, transferring all of its momentum and kinetic energy to the outgoing object.

During the collision in Newton's cradle we observe this. Shockwaves move through the balls and at the end of the process the end ball (which has the same mass as the first moving ball) moves away with virtually the same velocity as the inbound ball which has come to a complete standstill.

When repeated with two, three and four balls we observe each time that momentum and kinetic energy immediately before the collision is about equal to momentum and kinetic energy immediately after the collision, with the same number of balls moving away at nearly the same speed as those incoming. The slight loss of kinetic energy post-collision is due to the production of soundwaves and some very slight post-collision movement of the holding structure itself.

Before After

Bouncy Ball Collisions

Curriculum Links:

- Conservation of momentum
- Kinetic energy
- Elastic and inelastic collisions
- Energy transfer

Key Stage: 3 - 5

Equipment List:

- High bounce ball
- Clear hard floor space
- Deep tray or bowl of sand

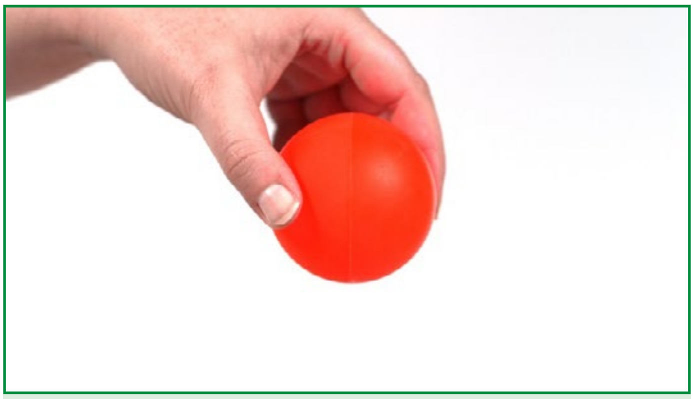
Procedure:

Lift the bouncy ball above the floor and note the height it is dropped from. Release the ball and observe the bounce, noting the height it reaches. Compare the two heights and get students to think about conservation of kinetic energy.

Now place the tray or bowl of sand under the ball. Repeat the previous procedure and observe the rebound height and what happens to the sand. Ask students to think about what has happened to the kinetic energy of the ball.

Expected Outcomes:

a) Dropping the ball onto the floor



Prior to release

Undergoes deformation during impact with the ground and rebounds

When the bouncy ball is dropped onto the hard floor from a height h_1, it rebounds up to almost to the same height, h_2.

Measuring these heights allows the loss of energy to be calculated:

Before collision:
Loss of PE falling from h_1 = KE just before collision
$mgh_1 = \frac{1}{2} mv_1^2$ so $v_1^2 = 2gh_1$

After Collision:
KE just after collision = gain in PE rising to height h_2
$\frac{1}{2} mv_2^2 = mgh_2$ so $v_2^2 = 2gh_2$

This gives $v_1^2 / v_2^2 = h_1/h_2$

This shows that as the starting and ending heights are nearly the same, the ball's rebound speed was almost the same as its impact speed. It lost hardly any kinetic energy during the collision and therefore is an example of a collision that is very nearly elastic.

b) Dropping the ball into sand

When the bouncy ball is dropped into the sand, it hardly rebounds at all. Instead, the kinetic energy of the ball is transferred into other forms including heat, sound and the kinetic energy of the ejected sand.

Momentum is still conserved; it is transferred to the grains of sand and ultimately to the Earth itself.

Collisions: Suggestions for detailed student tracker analysis.

Ground-based experiments
Coefficient of restitution measurement for bouncing ball: plotting maximum height per bounce.

Small-large sphere collisions: measuring velocities pre- and post-impact to map against theoretical values obtained from conservation of momentum and kinetic energy analysis.

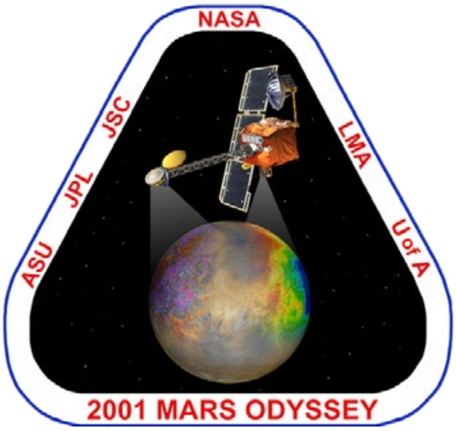
Tim's orbital demonstrations
Elastic collisions: Conducting velocity analyses (pre and post collision) in horizontal and vertical fields of view to check conservation of momentum in these two planes.

Inelastic collisions: Conducting pre- and post-collision velocity analyses with subsequent translational kinetic energy measurement. First-order modelling of rotational behaviour of combined assembly post-collision.

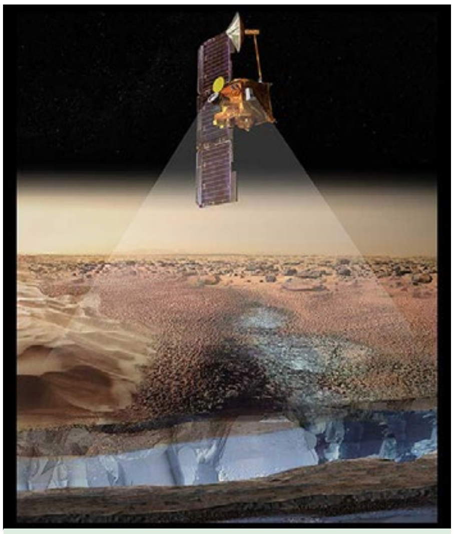
Collisions of objects of equal mass: Finding water on Mars

As we see from the theoretical analysis, ground experiments and Tim's orbital demonstrations, having a moving ball hitting a stationary ball of the same mass results in the maximum transfer of kinetic energy from impactor to target. To recap our modelling and experimentally obtained data:

- With balls of equal mass, the incoming ball (impactor) is brought to a standstill and the initially stationary ball moves off with maximal velocity.
- If the impactor ball has greater mass or less mass than the stationary target, then the impactor is not brought to a total standstill – and the stationary ball's subsequent post-collision velocity is not as much as in the case when the masses are equal.



This principle has been successfully utilised through the process of neutron spectroscopy to detect water hidden in the upper few metres of Martian regolith – the top layer of Mars' surface. Mars Odyssey, a NASA spacecraft which has been orbiting Mars since October 2001, carried a **neutron spectrometer** to measure the **neutron flux** produced due to the following processes:



Credit: NSA

- High-energy galactic cosmic radiation (GCR) hitting the Martian surface causes neutrons to be ejected from surface atomic nuclei
- Some of the ejected neutron flux is upwards towards the orbiting spacecraft which detects them
- Each chemical element in the surface creates its own unique distribution of neutron energies as different substances will absorb or slow down (moderate) the ORBIT HYDROGEN IONs, i.e. protons, HAVE THE SAME MASS AS THE NEUTRONS WHICH ARE MOVING THEY WILL HAVE THE GREATEST EFFECT IN MODERATING THE NEUTRONS.
- **Hydrogen in the soil (assumed to be in subsurface ice) will effectively absorb neutron energy and maximally reduce the neutron flux that escapes from the surface to be detected on orbit.**

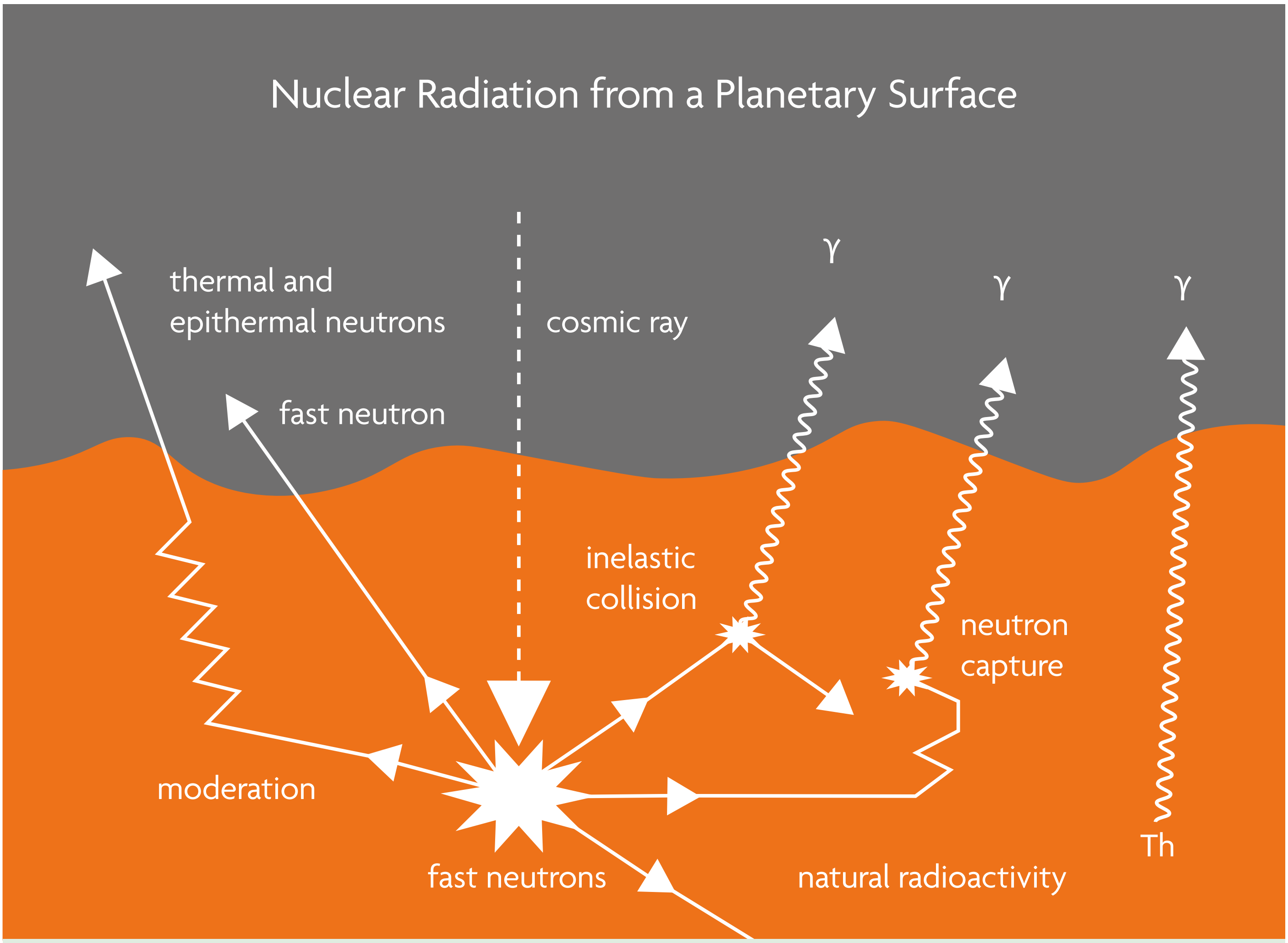
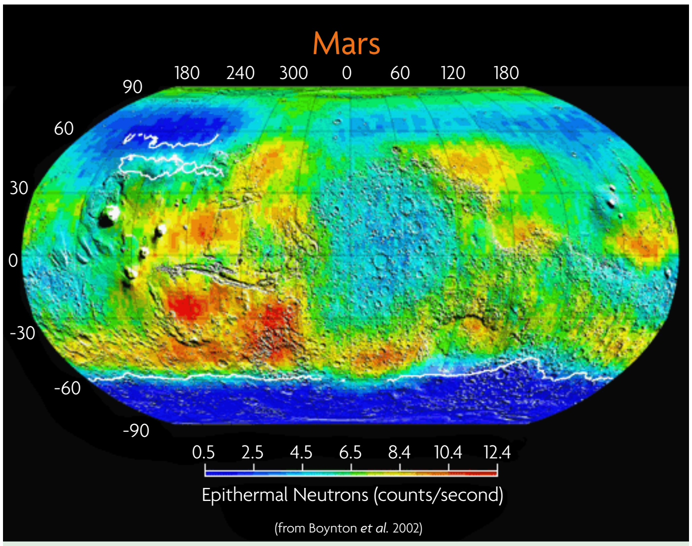


Image courtesy of NASA JPL

- The neutron flux “signatures” measured by the satellite thus allow determination of the top surface material’s chemical composition
- Additional gamma fluxes caused by nuclear de-excitation after GCR interactions with surface nuclei were also measured by the spacecraft



Neutron flux measured across the Martian surface

The following map shows the inferred distribution of subsurface ice from Mars Odyssey orbital data:

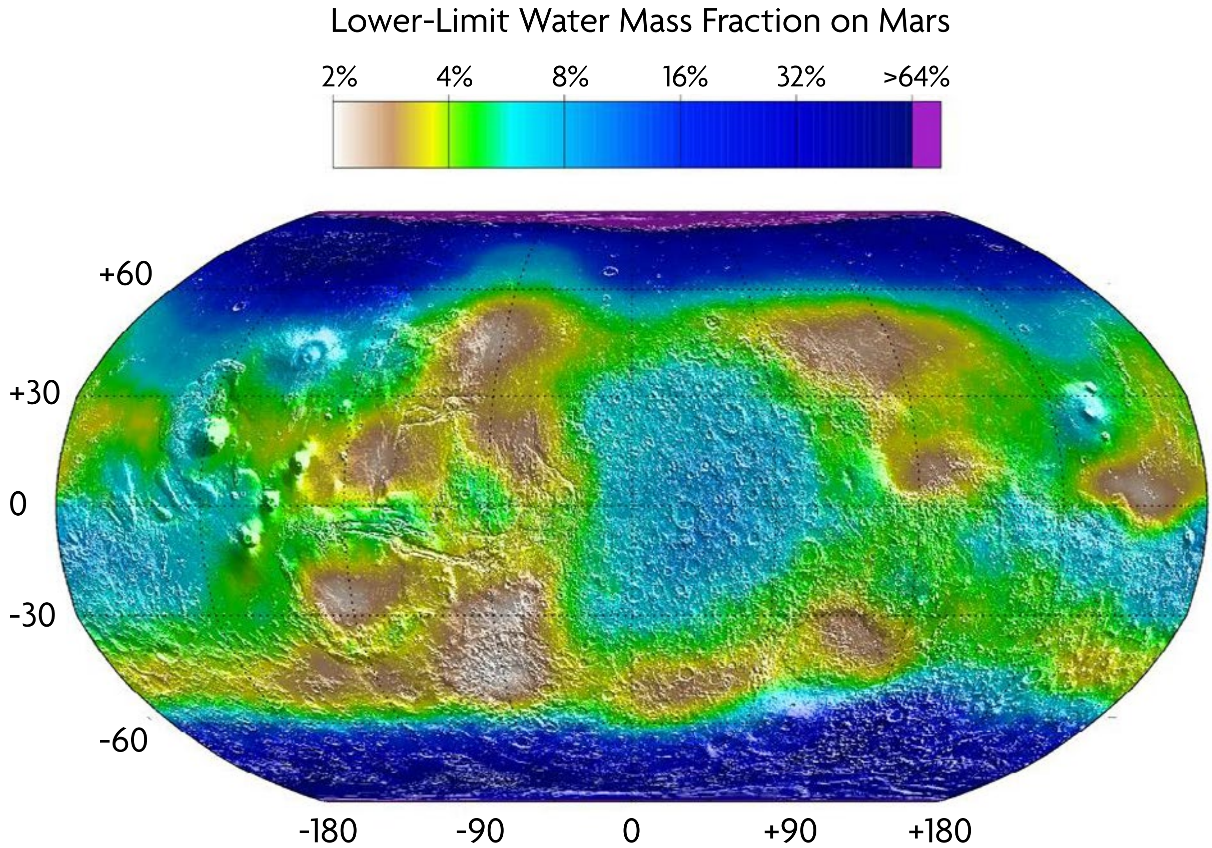


Image courtesy of NASA JPL

This dataset is valid for surface depths of only approximately 1m, but the large areas covered indicate enormous quantities of subsurface water exist in the upper few metres of the Martian surface.

Water on the Moon and Mercury

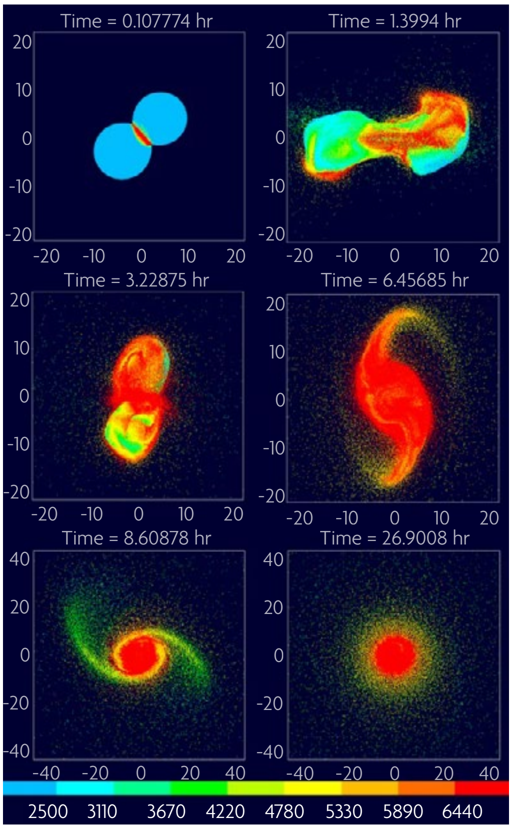
This same technique was used to confirm the presence of water on the Moon in polar regions by NASA's Lunar Prospector mission in 1998 and on Mercury by NASA's Messenger mission. Both these missions had neutron spectrometers aboard and the results strongly support the thesis that large quantities of ice may be present in the polar regions of both worlds.

Formation of Earth's Moon – the Theia impact

Computer modelling and geological evidence from the six Apollo lunar landings between 1969 and 1972 along with analysis of the Earth and Moon's current orbital, rotation states and isotopic analysis of Earth rocks suggest that the formation of Earth's Moon may have been caused by a collision with a Mars-sized planetary body called Theia. Perhaps 100 million years after the formation of the Earth, it is theorised that Theia struck the Earth at an oblique angle with a relative velocity of less than 4km/s. The primordial Earth's mantle material was blasted into orbit post-collision and accreted over a period of millions of years to form the Moon. During the collision it is theorised that the gain of angular momentum by the Earth resulted in a post-collision Earth day length of only a few hours (compared to the current 23 hours 56 minutes).

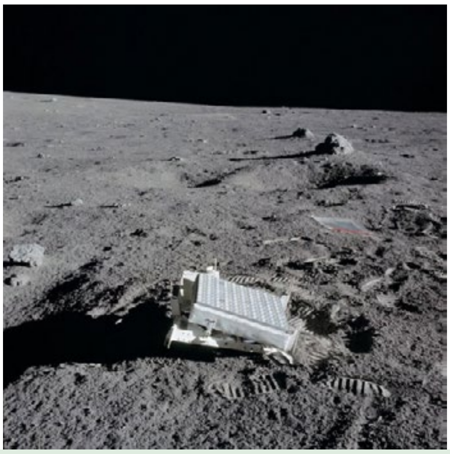
The images on the right show a simulation of an off-center, low-velocity collision between two protoplanets containing 45% and 55% of Earth's mass. The colours indicate particle temperature in kelvin, with a scale of blue-to-red representing temperatures from 2,000K to in excess of 6,440K. After the initial impact, the protoplanets re-collide, merge and form a rapidly spinning Earth-mass planet surrounded by an iron-poor protolunar disk containing about three lunar masses. The composition of the disk and the final planet's mantle differ by less than 1%.

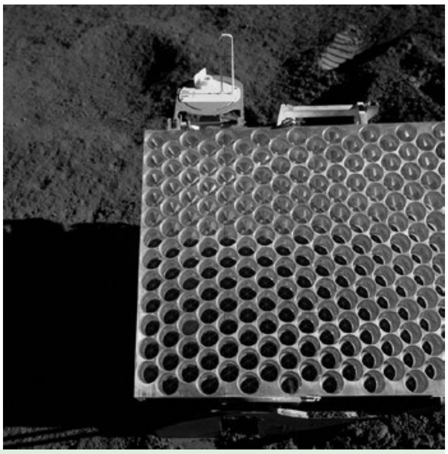
Credit: Southwest Research Institute- See more at: http://www.space.com/18106-moon-formation-earth-giant-impact.html#sthash.2VAg0dsR.dpuf



Post-impact gravitational interactions between the Earth and Moon have caused the eventual approximate coplanar nature of Earth's equator and the Moon's orbit around the Earth, whilst tidal interactions between the Earth and Moon have lengthened the length of Earth's day and increased the mean separation of Earth and Moon. The loss of Earth's rotational kinetic energy has manifested itself in the raising of the Moon in Earth's gravitational field – a process which continues now. From the Apollo lunar retroreflector laser experiments led by McDonald Observatory in Texas (see image, bottom right), it has been measured that every year the Moon recedes further from the Earth by just under 4cm per year.

McDonal Observatory, Texas

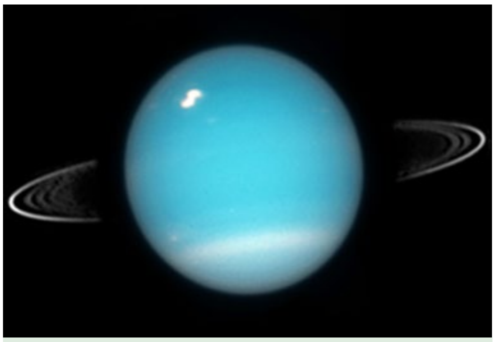




Retroreflectors left on the surface of the Moon Credit: NASA

Uranus' axial tilt

Uniquely among the Solar System's eight major planets, the ice/gas giant planet Uranus has an axial tilt of 98 degrees (compared, for example to the Earth's 23.5 degrees and Jupiter's 3 degrees). As clearly seen in this Hubble Space Telescope (HST) image, Uranus' rotational axis, along with the planes of orbits of its moons and rings, are dramatically tilted in relation to its orbital plane around the Sun.



Hubble image of Uranus

1. Launching a Rocket

When Tim Peake launched on his mission to the international space station the rocket used was a well tried and tested Soyuz-FG rocket.
The rocket and its payload had a combined mass of 307,100kg. In the initial stage of the launch the Soyuz FG was capable of generating a thrust of 793kN near the surface for 118s.

(a) Calculate the impulse of the rocket

(b) What is the momentum change of the rocket?

(c) Calculate the change in speed of the rocket (assuming its mass doesn't change in this time)

In reality of course the rocket is expelling exhaust gases and therefore reducing in mass during the launch.

www.esa.int

It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$v = v_e \ln(m_0/m_r)$

Where v_e is the velocity of the exhaust gases ejected from the rocket boosters and $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass (m_0) of the rocket to what is left (m_r) after all of the fuel is exhausted.

(d) Calculate the mass ratio needed to escape the Earth's gravity from rest, given that the escape velocity from Earth is about 11200ms^{-1} and assuming the rockets exhaust velocity was 2500 ms^{-1}.

The calculation will show that only 1/88 of the mass is left when the fuel is burnt, and 87/88 of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%.

Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$.

It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favourable too.

2. Docking with the International Space Station

The Soyuz-TMA module carrying Tim Peake had a mass of 7150kg and docked with the International Space Station which has a mass of 370,000kg. Assume that the relative speed before docking is $0.20 ms^{-1}$ and a successful 'soft' docking is achieved first time.

en.wikipedia.org

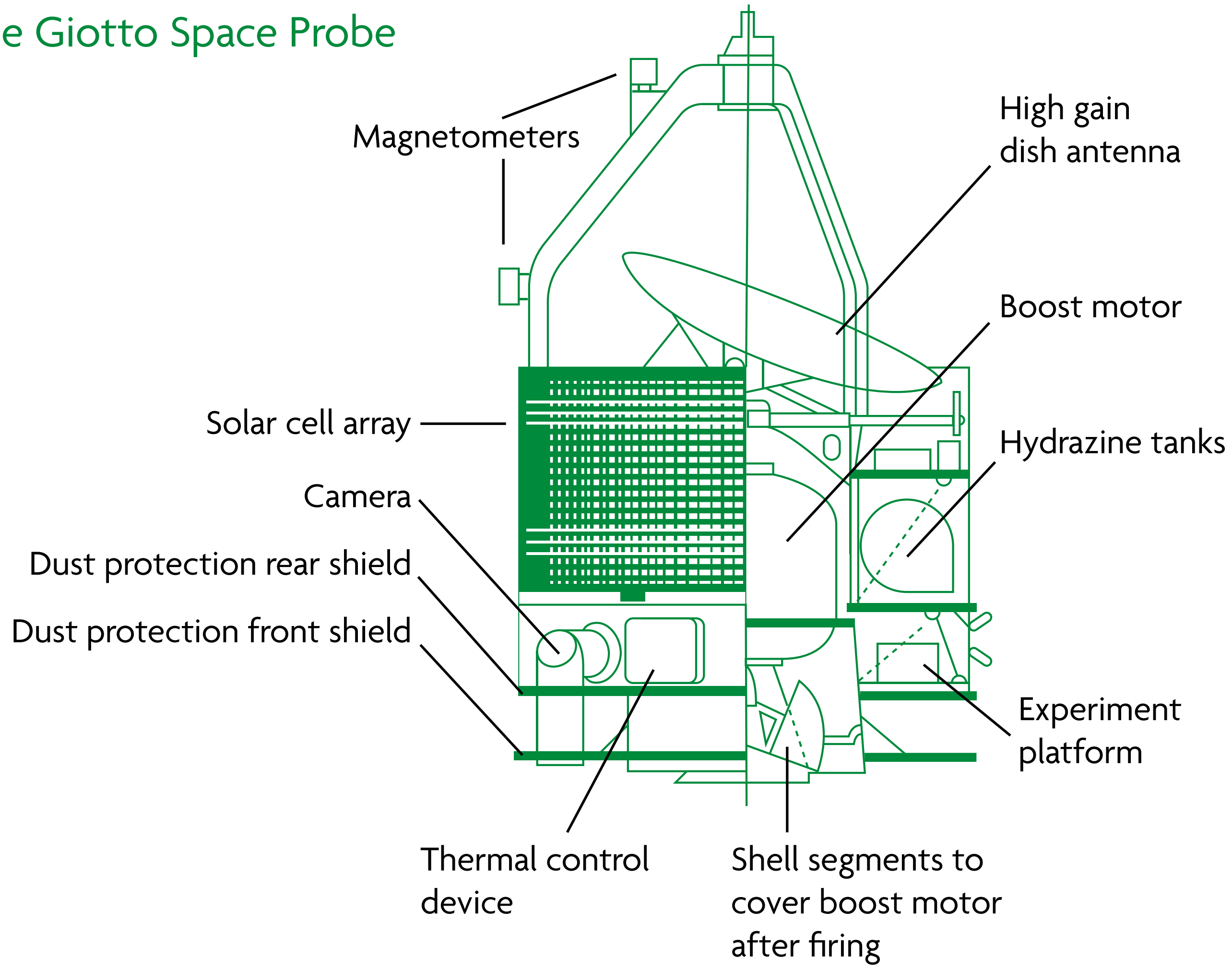
(a) Calculate how much faster the combined object is travelling than the original speed of the International Space Station.

(b) The International Space Station (ISS) is travelling at an orbital speed of $7.660 kms^{-1}$. If it was hit head on by a 10kg meteorite travelling at $50 kms^{-1}$, which embedded in the ISS on impact, calculate the speed by which the ISS will be slowed down.

3. Giotto

In June 1999, the ESA space probe Giotto made an Earth fly-by following missions to investigate Halley's Comet in 1986 and Comet Grigg-Skjellerup in 1992. A major hazard to Giotto was the large number of high-speed solid particles ('dust') that make up comets' tails. At collision speeds likely to occur, a 0.1g particle can penetrate an aluminium plate 8cm thick. To protect the probe's instruments, engineers designed a dust shield of two protective sheets 23cm apart. The front shield is a sheet of aluminium 1mm thick which retards and vaporises all but the largest particles. The rear shield is a 12mm thick sheet of Kevlar (as used in bullet-proof vests) which traps any remaining debris and becomes heated as a result.

The Giotto Space Probe



The Giotto probe has a mass of 960kg. Suppose it is travelling at 2.0kms^{-1} when it encounters a 'dust' particle of mass 0.10g travelling at 50kms^{-1} in the opposite direction to the probe. The particle is trapped in the shield.

(a) Show that the collision has a negligible effect on Giotto's velocity

(b) If the collision takes 1.0ms, calculate the average force exerted on the probe

(c) Explain through calculation whether this is an elastic or an inelastic collision

4. Lunar Craters

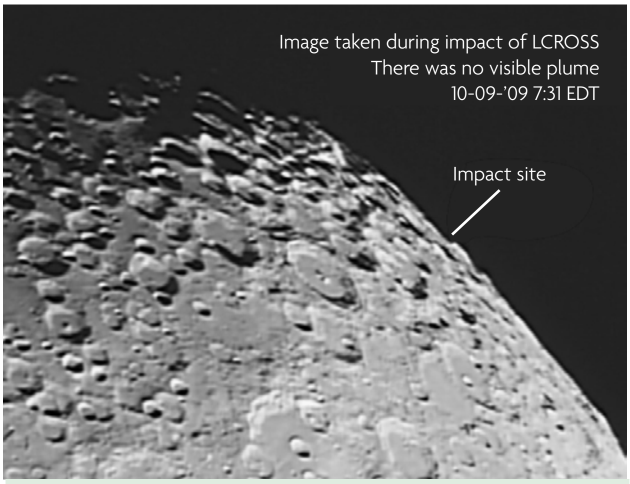
The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of 5×10^{12}kg (about a kilometre across) strikes the Moon at a speed of 15.0kms^{-1}.

(a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is 7.36×10^{22}kg)?

(b) How much kinetic energy is lost in the collision?

Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon.

(c) In October 2009, NASA crashed a rocket into the Moon (right), and analysed the plume produced by the impact (significant amounts of water were detected.)



LCROSS impact. Credit: NASA

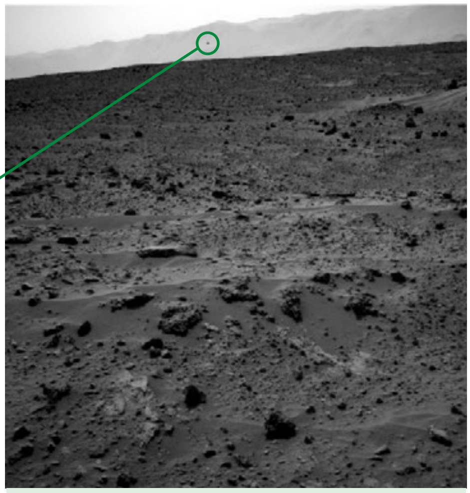
Answer part (a) and (b) again, but this time for this real-life experiment. The mass of the rocket was 2000kg and its speed upon impact was 9000km per hour^{-1}.

5. Flying in Planetary Atmospheres

A kestrel is a bird of prey which is commonly seen hovering alongside motorway verges looking for prey. To remain in flight the bird has to push down on the air so that an equal and opposite force can be generated (according to Newton's 3rd Law) which is large enough to balance the bird's own weight.

(a) By considering the relationship between force and momentum F=dp/dt determine the downward speed given to the air by the kestrel. The kestrel has a body mass of 200g and pushes down a column of air of area 600cm^2. (Density of air = 1.3kgm^{-3}, assume g = 9.8ms^{-2})

(b) Estimate the minimum power that the kestrel needs to hover

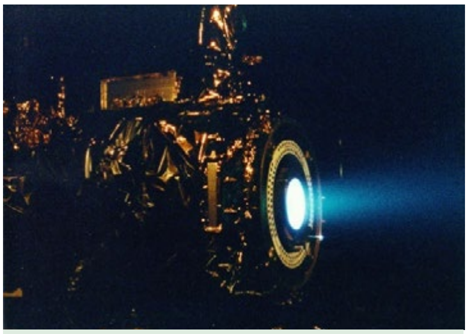


opoikiyo.blogspot.com

In 2014 it was reported that UFO enthusiast website UFO-Blogger claimed that this photograph taken by NASA's Curiosity Rover shows a kestrel-like bird hovering above Mars surface!

(c) If the minimum power needed for the kestrel to hover above the Martian atmosphere is 20W determine the density of the Martian atmosphere (we will assume that our kestrel has a supply of oxygen!). For the Martian surface assume g= 3.8 ms^{-2}.

6. Ion Drives



Deep Space 1. Credit: NASA

Deep Space 1 (pictured) was the first NASA spacecraft to use an ion drive propulsion system. A beam of xenon ions (charged atoms) was fired backwards, propelling the spacecraft forwards.

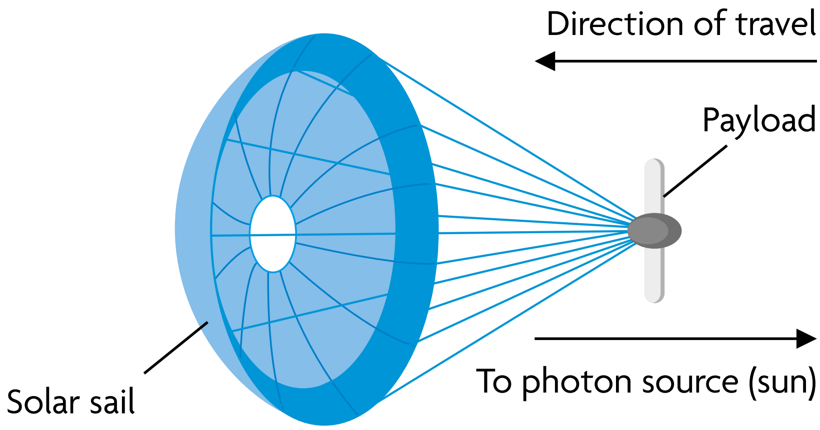
(a) The mass of a xenon ion is $2.2\text{x}10^{-25}$kg, and it was ejected at a speed of $3.1\text{x}10^{4}\text{ms}^{-1}$. Calculate the number of ions that would have to be emitted per second to generate a thrust of $92 \text{ x } 10^{-3}$N.

(b) Deep Space 1 carried about 80kg of reaction mass. For how long could the engine continue supplying this amount of thrust before running out of reaction mass to shove out the back?

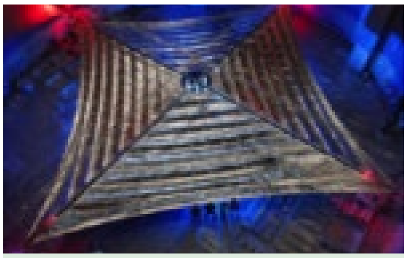
(c) Estimate the amount of xenon that would have to be ejected from the drive each second to generate the same thrust as the Soyuz-FG rocket used to launch Tim Peake (i.e. 793kN)

7. Solar Sail Challenge

As a solar sail, LightSail-1's propulsion is dependent on solar radiation alone. In-falling photons will exert radiation pressure on the sail, producing a small degree of acceleration. Thus, the solar sail will be propelled by pressure from sunlight itself, and not by the charged particles of the solar wind. Although the acceleration is expected to be slow, it will be continuous, permitting LightSail-1 to reach relatively high speeds over time.



LightSail-1 has four triangular sails, which combine to form a rectangular-shaped surface. The spacecraft will have a 3U (3-litre volume) CubeSat format, and will unfold to its full size upon deployment at its designated orbit altitude of 500 miles (800km). The sails are made of Mylar, a reflective polyester film, and they have a total area of 32m^2, which would make the craft easily visible to naked-eye observers on Earth. The mass of Lightsail-1 is approximately 4.5kg.



news.nationalgeographic.com

a) Estimate the force on the Lightsail-1 solar sail due to the photons from the sun reflecting from its surface when in orbit close to the Earth (hint: the energy of a photon is given by $E = hf$ and the momentum of a photon is given by $p = h/\lambda$)

b) Calculate the area of a solar sail that would be needed to generate the same thrust as the Soyuz-FG rocket that launched Tim Peake (i.e. 793kN)

Useful Data

Luminosity of Sun (Power Output) = 3.85×10^{26}W
$h = 6.63 \times 10^{-34}$Js
Wavelength Range of visible light = 400-700nm

Planet	Distance from Sun (in millions of kilometers)
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	778
Saturn	1,427
Uranus	2,871
Neptune	4,497
Pluto	5,914

8. Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship.

Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time.

Among the things to be considered are the masses involved, the force she can exert on the packages over set distances, and the distance to the ship.

Bruce McCandless in 1984 using a Manned Maneuvering Unit or MMU en.wikipedia.org

1. Launching a Rocket

When Tim Peake launched on his mission to the international space station the rocket used was a well tried and tested Soyuz-FG rocket.

The rocket and its payload had a combined mass of 307,100kg. In the initial stage of the launch the Soyuz FG was capable of generating a thrust of 793kN near the surface for 118s.

(a) Calculate the impulse of the rocket

Impulse = $793 \times 10^3 \times 118 = 9.36 \times 10^7$ Ns (to 3 s.f.)

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(b) What is the momentum change of the rocket?

Impulse = change of momentum = $9.36 \times 10^7 kgms^{-1}$

(c) Calculate the change in speed of the rocket (assuming its mass doesn't change in this time)

$\Delta p = m\Delta v$

$\Delta v = 9.36 \times 10^7 / 307{,}100 = 305ms^{-1}$ (to 3 s.f.)

In reality of course the rocket is expelling exhaust gases and therefore reducing in mass during the launch.

It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$V = V_e \ln(m_0/m_r)$

Where v_e is the velocity of the exhaust gases ejected from the rocket boosters and $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass (m_0) of the rocket to what is left (m_r) after all of the fuel is exhausted.

(d) Calculate the mass ratio needed to escape the Earth's gravity from rest, given that the escape velocity from Earth is about 11200ms^{-1} and assuming the rockets exhaust velocity was 2500ms^{-1}.

$11{,}200 = 2500 \ln (m_0/m_r)$

$m_0/m_r = e^{11{,}200/2500} = 88.2$

This result means that only 1/88 of the mass is left when the fuel is burnt, and 87/88 of the

initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%.
Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$.

It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favourable, too.

2. Docking with the International Space Station

en.wikipedia.org

The Soyuz-TMA module carrying Tim Peake had a mass of 7150kg and docked with the International Space Station of mass of 370,000kg. Assume that the relative speed before docking is $0.20 ms^{-1}$ and a successful 'soft' docking is achieved first time.

(a) Calculate how much faster the combined object is travelling than the original speed of the International Space Station.
Consider ideas of conservation of momentum
$7150 \times 0.20 = (370,000 + 7150) \times v$
$v = 3.79 \times 10^{-3} ms^{-1}$

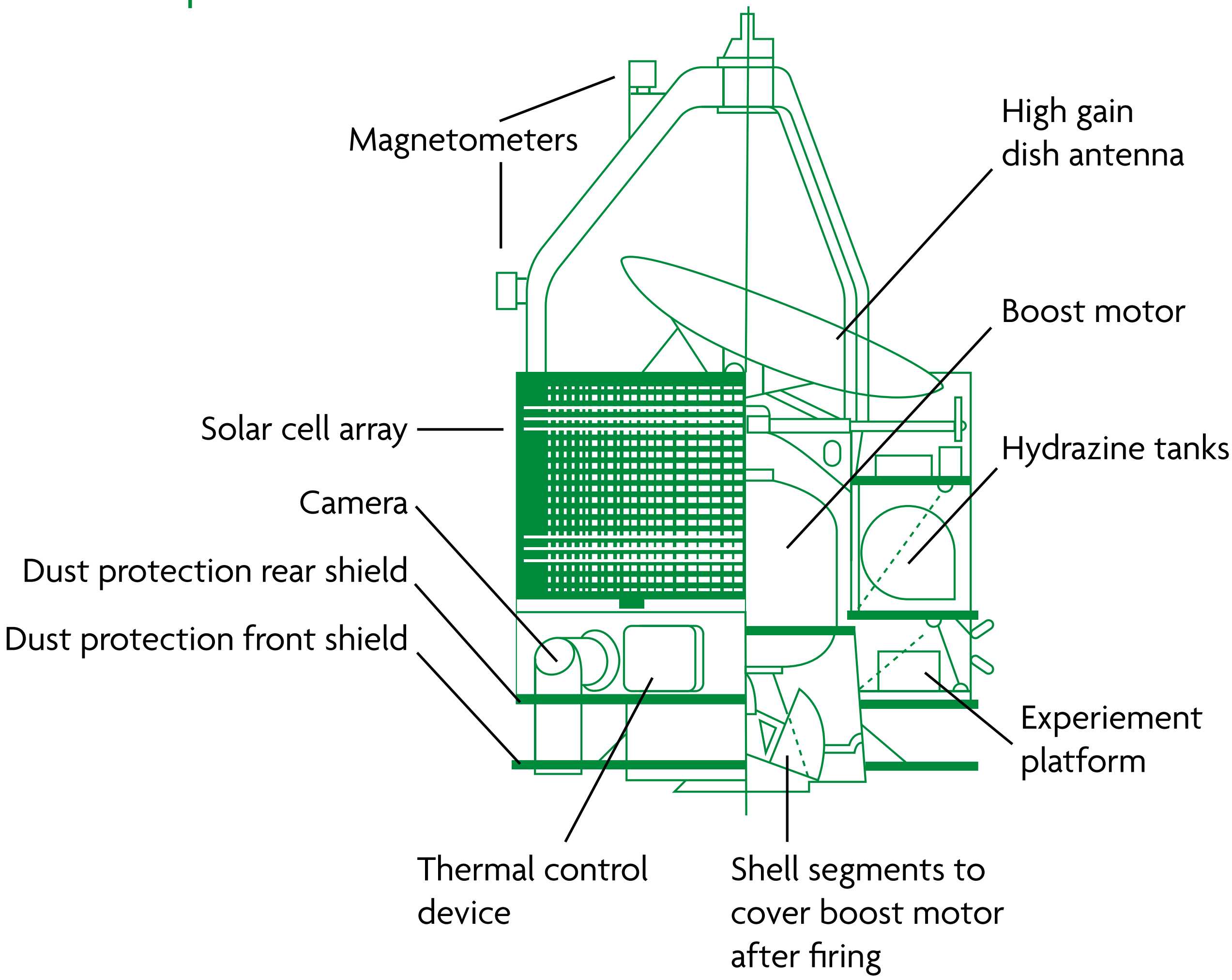
(b) The international space station (ISS) is travelling at an orbital speed of $7.660 kms^{-1}$. If it was hit head on by a 10.00kg meteorite travelling at $50.0 kms^{-1}$, which embedded in the ISS on impact, calculate the speed by which the ISS will be slowed down.
$370,000 \times 7.66 \times 10^3 - 10 \times 50 \times 10^3 = 370,010 \times v$
$v = 7658.4 ms^{-1}$ so $\Delta v = 1.6 ms^{-1}$

3. Giotto

In June 1999, the ESA space probe Giotto made an Earth fly-by following missions to investigate Halley's Comet in 1986 and Comet Grigg-Skjellerup in 1992. A major hazard to Giotto was the large number of high-speed solid particles ('dust') that make up comets' tails. At collision speeds likely to occur, a 0.1g particle can penetrate an aluminium plate 8cm thick. To protect the probe's instruments, engineers designed a dust shield of two protective sheets 23cm apart. The front shield is a sheet of aluminium 1mm thick which retards and vaporises all but the largest particles. The rear shield is a 12mm thick sheet of Kevlar (as used in bullet-proof vests) which traps any remaining debris and becomes heated as a result.

The Giotto probe has a mass of 960kg. Suppose it is travelling at $2.0 kms^{-1}$ when it encounters a 'dust' particle of mass 0.10g travelling at $50 kms^{-1}$ in the opposite direction to the probe. The

The Giotto Space Probe



(a) Show that the collision has a negligible effect on Giotto's velocity

$960 \times 2 \times 10^3 - 0.1 \times 10^{-3} \times 50 \times 10^3 = (960 + 0.1 \times 10\text{-}3) \times v$

$v = 2000 ms^{-1}$ (to 2 s.f.)

(b) If the collision takes 1.0 ms, calculate the average force exerted on the probe

$F = \Delta mv/\Delta t = (0.1 \times 10^{-3} \times 50 \times 10^3)/1 \times 10^{-3} = 5{,}000N$

(c) Explain through calculation whether this is an elastic or an inelastic collision

KE before $= 0.5 \times 960 \times 2000^2 + 0.5 \times 0.1 \times 10^{-3} \times 50{,}000^2$

$= 1.92 \times 10^9 + 125{,}000$

$= 1.92 \times 10^9 J$

KE after $= 0.5 \times (960 + 0.1 \times 10^{-3}) \times 2000^2$

$= 1.92 \times 10^9 J$

Working to 3 s.f. this collision appears to be elastic! But the particle has lost all kinetic energy on impact and undergoes a perfectly inelastic collision

4. Lunar Craters

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of 5.00×10^{12}kg (about a kilometre across) strikes the Moon at a speed of $15.0kms^{-1}$.

(a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is 7.36×10^{22}kg)?

$5 \times 10^{12} \times 15 \times 10^{3} = (7.36 \times 10^{22} + 5 \times 10^{12}) \times v$

$v = 1.02\times10^{-6}ms^{-1}$

(b) How much kinetic energy is lost in the collision?

$KE = ½ mv^2 = 0.5 \times 5 \times 10^{12} \times (15 \times 10^{3})^2 = 5.625 \times 10^{20}$ J assuming moon has gained negligible KE

Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon.

(c) In October 2009, NASA crashed a rocket into the Moon (right), and analysed the plume produced by the impact (significant amounts of water were detected.)

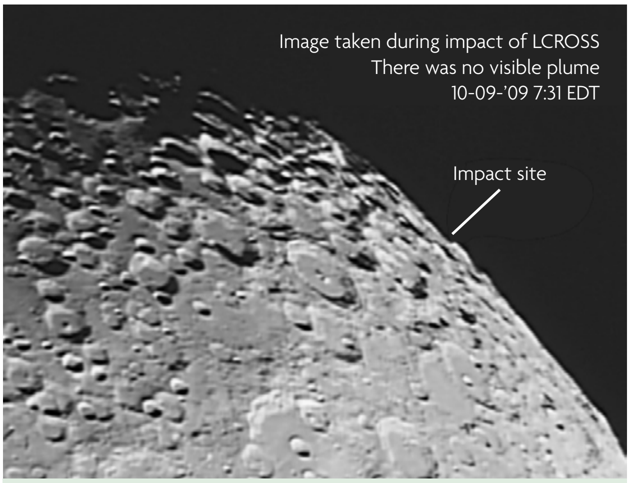
Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000kg and its speed upon impact was 9000 km/hour^{-1}.

$9000\ km/hour^{-1} = (9000 \times 10^{3}) / 3600 = 2500ms^{-1}$

$2000 \times 2500 = (7.36 \times 10^{22} + 2000) \times v$

$v = 6.8 \times 10^{-17}\ ms^{-1}$

$KE = ½ mv^2 = 0.5 \times 2000 \times (2500)^2 = 6.25\times10^{9}$ J



LCROSS impact. Credit: NASA

5. Flying in Planetary Atmospheres

A kestrel is a bird of prey which is commonly seen hovering alongside motorway verges looking for prey. To remain in flight the bird has to push down on the air so that an equal and opposite force can be generated (according to Newton's 3rd Law) which is large enough to balance the bird's own weight.

(a) By considering the relationship between force and momentum F=dp/dt determine the downward speed given to the air by the kestrel. The kestrel has a body mass of 200g and pushes down a column of air of area $600cm^2$.
(Density of air=$1.3kgm^{-3}$, take $g=9.8ms^{-2}$)

$F = \Delta mv/\Delta t = v\Delta m/\Delta t$

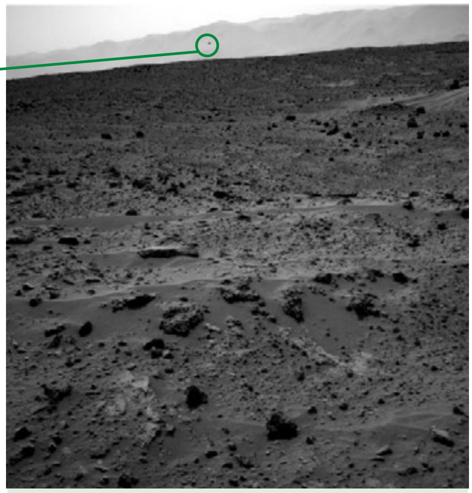
But $\Delta m/\Delta t = \rho A v$ so $F = \rho A v^2 = mg$ (as the lift is balancing the bird's weight)

$v = ((0.2 \times 9.81) / (1.3 \times 600/100^2))^{1/2} = 5.02 ms^{-1}$

(b) Estimate the minimum power that the kestrel needs to hover

$P=Fv = mgv = 0.2 \times 9.81 \times 5.02 = 9.85W$

In 2014 it was reported that UFO enthusiast website UFO-Blogger claimed that this photograph taken by NASA's Curiosity Rover shows a kestrel-like bird hovering above Mars surface!



opoikiyo.blogspot.com

(c) If the minimum power needed for the kestrel to hover above the Martian atmosphere is 20W determine what the density of the Martian atmosphere is (we will assume that our kestrel has a supply of oxygen!) On the Martian surface take $g = 3.8ms^{-2}$.

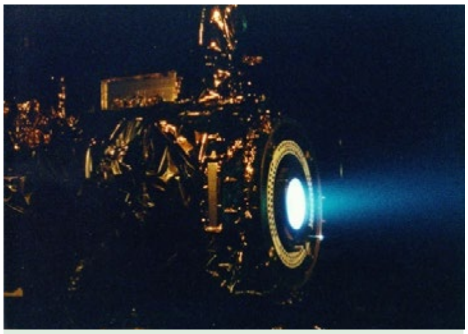
$P = Fv$

$v = 20/(0.2 \times 3.8) = 26.3ms^{-1}$

$\rho = mg/Av^2 = (0.2 \times 3.8) / ((600/100^2) \times 26.3^2) = 0.018kgm^{-3}$

6. Ion Drives

Deep Space 1 (pictured) was the first NASA spacecraft to use an ion drive propulsion system. A beam of xenon ions (charged atoms) was fired backwards, propelling the spacecraft forwards.



Deep Space 1. Credit: NASA

(a) The mass of a xenon ion is 2.2×10^{-25}kg, and it was ejected at a speed of $3.1 \times 10^4 ms^{-1}$. Calculate the number of ions that would have to be emitted per second to generate a thrust of 92×10^{-3}N.

$F=\Delta mv/\Delta t = 2.2 \times 10^{-25} \times 3.1 \times 10^4 = 6.82 \times 10^{-21}N$

number of ions per second = $92 \times 10^{-3} / 6.82 \times 10^{-21} = 1.35 \times 10^{19}$ ions

alternative approach $F=v\Delta m/\Delta t$ $92 \times 10^{-3} = 3.1 \times 10^4 \Delta m/\Delta t$ so $\Delta m/\Delta t = 2.968 \times 10^{-6} kgs^{-1}$

$2.968 \times 10^{-6}/2.2 \times 10^{-25} = 1.35 \times 10^{19}$ ions

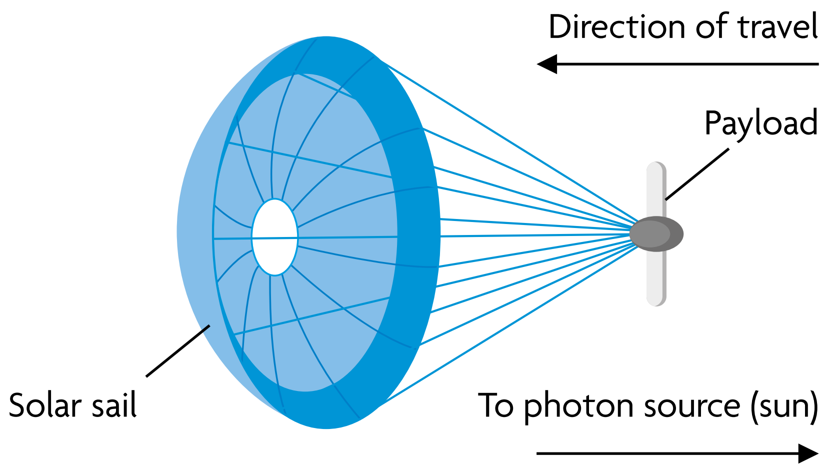
(b) Deep Space 1 carried about 80kg of reaction mass. For how long could the engine continue supplying this amount of thrust before running out of reaction mass to shove out the back?
mass of gas emitted per second = $1.35 \times 10^{19} \times 2.2 \times 10^{-25} = 2.87 \times 10^{-6}$ kgs^{-1}

$80 / 2.87 \times 10^{-6} = 2.79 \times 10^{7}$s about 323 days

(c) Estimate the amount of xenon that would have to be ejected from the drive each second to generate the same thrust as the Soyuz-FG rocket used to launch Tim Peake (i.e. 793kN)
$F=\Delta mv/\Delta t$ so $\Delta m = F\Delta t/v = 793 \times 10^{3} \times 1 / 3.1 \times 10^{4} = 25.6$kg

7. Solar Sail Challenge

As a solar sail, LightSail-1's propulsion is dependent on solar radiation alone. In falling photons will exert radiation pressure on the sail, producing a small degree of acceleration. Thus, the solar sail will be propelled by pressure from sunlight itself, and not by the charged particles of the solar wind. Although the acceleration is expected to be slow, it will be continuous, permitting LightSail-1 to reach relatively high speeds over time.



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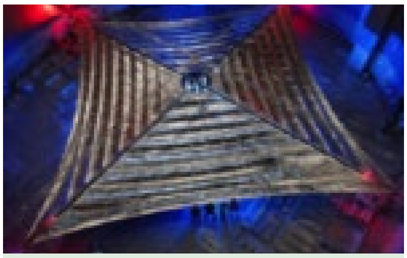
news.nationalgeographic.com

a) Estimate the force on the Lightsail-1 solar sail due to the photons from the sun reflecting from its surface when in orbit close to the Earth (hint the energy of a photon E=hf and the momentum of a photon p=h/λ)

b) Calculate the area of solar sail that would be needed to generate the same thrust at the Soyuz-FG rocket that launched Tim Peake (i.e. 793kN)

Useful Data

Luminosity of Sun (Power Output) = 3.85×10^{26} W
h = 6.63×10^{-34} Js
Wavelength Range of visible light = 400-700 nm

Planet	Distance from Sun (in millions of kilometers)
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	778
Saturn	1,427
Uranus	2,871
Neptune	4,497
Pluto	5,914

Pick an average wavelength = 550nm

Energy of one photon = hf = hc/λ = $(6.63 \times 10^{-34} \times 3 \times 10^{8}) / 550 \times 10^{-9}$

Energy of 1 photon = 3.62×10^{-19} J

Energy arriving at Earth per square meter per second = $3.85 \times 10^{26} / (4\pi \times (1.5 \times 10^{11})^2) = 1361 \text{Wm}^{-2}$

Number of photons arriving per square meter per second = $1361 / 3.62 \times 10^{-19} = 3.76 \times 10^{21}$

Number of photons hitting solar sail per second = $3.76 \times 10^{21} \times 32 = 1.23 \times 10^{23}$

Momentum of one photon p=h/λ = $6.63 \times 10^{-34} / 550 \times 10^{-9} = 1.21 \times 10^{-27} \text{kgms}^{-1}$

Change in momentum when photon reflects = 2p

Force = Change in Momentum/second

Force = number of photons hitting sail per second x 2p
= $1.23 \times 10^{23} \times 2 \times 1.21 \times 10^{-27}$
= 2.9×10^{-4} N

Force on 32 m^2 lightsail = 2.9×10^{-4} N

Force on Soyuz = 793×10^3 N

Area needs to be = $793 \times 10^3 / 2.9 \times 10^{-4}$ times bigger = $2.7 \times 10^9 \times 32 = 8.75 \times 10^{10} m^2$

8. Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship.

Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time.

Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

Bruce McCandless in 1984 using a Manned Maneuvering Unit or MMU en.wikipedia.org